A Practical Approach for Wall Shear Stress Topological Skeleton Analysis Applied to Intracranial Aneurysm Hemodynamics

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Abstract— **The physiopathological role of Wall Shear Stress (WSS) in intracranial aneurysm development/rupture and the action of contraction/expansion played by shear forces on vessel wall make topological skeleton analysis of the WSS vector field of great interest. Here we present a practical way to analyse WSS topological skeleton through the identification and classification of WSS fixed points and manifolds. The method is based on the calculation of the WSS vector field divergence and Poincarè index, and it is here successfully applied to a dataset computational hemodynamic models of intracranial aneurysms.**

Keywords—**Wall shear stress, topological skeleton, divergence.**

I. Introduction

A large body of literature has demonstrated the importance of WSS in the onset and progression of cardiovascular diseases [1]. In particular, WSS has been proposed as an indicator of rupture risk in intracranial aneurysms, characterized by a complex hemodynamics. However, the ability of WSS-based indicators to predict aneurysm wall pathophysiology and remodelling, and to discriminate aneurysms rupture risk, is still debated due to large variations and frequent contradictory results from different studies [2]. An in-depth analysis is needed, to close the gap of knowledge currently limiting the use of WSS as a biomarker for diagnostic and prognostic purposes.

In this context, recent studies have highlighted the relevance of WSS fixed points, and the stable and unstable manifolds that connect them [3]. The relevance of these WSS topological features lies in their ability to highlight the complex and highly dynamic features of the WSS field, as well as their strong link with flow features like flow stagnation, separation, and recirculation. Technically, a fixed point of a vector field is a point where the vector field vanishes, while unstable/stable vector field manifolds identify contraction/expansion regions linking fixed points. The presence of WSS fixed points and of WSS contraction/expansion regions, highlighted by WSS manifolds, might induce focal vascular responses relevant for aneurysm rupture [3]. For these reasons, the topological skeleton analysis of the WSS vector field is of great interest and motivates the study present herein.

Lagrangian-based approaches have been recently proposed to identify WSS manifolds but have certain practical limitations [4]. A Eulerian approach has also been suggested, but only for 2D analytical fields [5]. Here we propose and demonstrate the use of a simple Eulerian approach for identifying WSS topological skeleton on 3D surfaces. The proposed method is applied to eight personalized computational hemodynamic models of intracranial aneurysm.

II. Methods

A. Computational Hemodynamics

Eight ICA aneurysm computational hemodynamics models enrolled in a broader study of 3D phase contrast MRI (PC-MRI) were considered. Details on geometry reconstruction, personalized conditions at boundaries and CFD simulations are reported elsewhere [6].

B. Practical way for WSS topological skeleton identification

The Volume Contraction theory demonstrates that the divergence of vector fields gives practical information on the associated dynamical systems, avoiding numerical integration for manifolds identification (as required by Lagrangian-based approaches), thus reducing the computational effort. In particular, the divergence is able to (1) encase stable/unstable manifolds, and (2) identify the basins of attractions of each attractor. For those reasons, a divergence-based approach for WSS manifolds identification at the luminal surface of intracranial aneurysms is proposed. As WSS divergence depends upon the algebraic summation of the magnitude of single gradients of WSS vector components, it might fail in properly identifying expansion/contraction regions. To overcome this limitation, here the divergence of the normalized WSS vector field was considered:

$$\nabla \cdot (\boldsymbol{\tau}_{\mathbf{u}}) = \nabla \cdot \left(\frac{\boldsymbol{\tau}}{\|\boldsymbol{\tau}\|_2}\right), \tag{1}$$

where $\boldsymbol{\tau}_{\mathbf{u}}$ is the WSS unit vector. Eq. (1), neglecting the vector field magnitude variation but taking into account variation of directions of the vector field only, correctly identifies WSS manifolds and therefore is suitable for practical WSS topological analysis. To complete the analysis, we propose a robust method for WSS fixed points identification. The Poincarè index is considered here for WSS fixed points identification because of its mesh-independent and topologically invariant proprieties. Then, a Jacobian analysis of WSS fixed points allows to classify the fixed point attractive or repelling nature. The proposed practical approach for the WSS topological skeleton identification is applied to both cycle-average WSS vector field $\bar{\boldsymbol{\tau}}(\boldsymbol{x})$:

$$\bar{\boldsymbol{\tau}}(\boldsymbol{x}) = \frac{1}{T}\int_0^T \boldsymbol{\tau}(t, \boldsymbol{x})dt \tag{2}$$

where T is the cardiac cycle duration, and instantaneous WSS vector fields at the luminal surface. It was previously suggested that cycle-average WSS vector field $\bar{\boldsymbol{\tau}}$ fixed points and their associated manifolds influence the near-wall intravascular transport [4]. However, it is clear from Eq. (2)

that it could be possible by construction that a $\bar{\boldsymbol{\tau}}$ fixed point would have never been a real WSS fixed point all along the cardiac cycle, thus stimulating a more in-depth investigation focusing on the time dependence of WSS fixed points and manifolds location and nature along the cardiac cycle.

Hence, WSS fixed points analysis is applied here to instantaneous WSS vector field and a measure to quantify the fraction of cardiac cycle spent by instantaneous WSS fixed points at a specific location at the luminal surface is proposed:

$$RT_{x_{fp}}(e) = \frac{\bar{A}}{A_e}\frac{1}{T}\int_0^T \mathbb{I}_e\left(\boldsymbol{x}_{fp}, t\right)\, dt \qquad (4)$$

where $\boldsymbol{x}_{\boldsymbol{fp}}(t)$ is the WSS fixed point position at time $t \in [0, T]$, e is the generic triangular element of the superficial mesh of area A_e, $\bar{A}$ the average surface area of all triangular elements of the superficial mesh and $\mathbb{I}$ is the indicator function.

Moreover, a modified version of Eq. (4) is proposed here, where the residence time of a fixed point is weighted by the absolute value of the instantaneous WSS vector divergence, intended as a measure of the strength of the local contraction/expansion action of local shear forces:

$$RT\nabla_{x_{fp}}(e) = \frac{\bar{A}}{A_e}\frac{1}{T}\int_0^T \mathbb{I}_e\left(\boldsymbol{x}_{fp}, t\right) \left|(\nabla \cdot \boldsymbol{\tau})_e\right| dt \qquad (5)$$

where $(\nabla \cdot \boldsymbol{\tau})_e$ is the instantaneous WSS divergence computed in the surface triangle e containing the instantaneous WSS fixed point.

III. Results

A. Analysis of the Cycle-Average WSS Vector Field

An analytical vector field was used for benchmarking purposes. The proposed method was compared to the classical vector field integration approach [7] and to the recent trajectory-free method [5], providing excellent results. Then, the cycle-average WSS vector field at the luminal surface of the 8 intracranial aneurysm models was analyzed. The cycle-average WSS topological skeleton of one explanatory intracranial aneurysm model is presented in Figure 1.

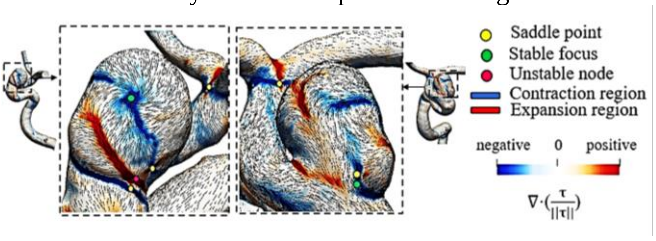

Figure 1: Topological skeleton of cycle-average WSS vector.

The contraction and expansion patterns, identifying unstable and stable manifolds, represent the basins of attraction for the stable fixed points associated with the manifolds. Notably, all cycle-average WSS stable fixed points at the luminal surface of the 8 intracranial aneurysm models were located within contraction regions, while the unstable node identified was located within an expansion region, thus confirming the appropriateness of the proposed method.

B. Analysis of instantaneous WSS vector field

For an in-depth characterization of the WSS fixed points, quantities $RT_{x_{fp}}(e)$ and $RT\nabla_{x_{fp}}(e)$ were computed on the surface of all models and an explanatory example is presented in Figure 2. For visualization purposes, regions of interest R_{fp} were identified at the luminal surface around high $RT_{x_{fp}}(e)$ areas and including the identified cycle-average WSS fixed points locations (labeled from A_A to G_A). For the explanatory model, the results of the WSS fixed points residence times analysis clearly show that: (1) in regions R_{E_A} and R_{D_A} fixed points residence times were up to 70% of the cardiac cycle; (2) instantaneous WSS fixed points resided for moderate fractions of the cardiac cycle (range 3.6-58.2%) in cycle-average WSS fixed points identified locations; (3) the instantaneous WSS fixed points are always of the same nature as cycle-average WSS fixed points; (4) elevated values of the strength of the expansion/contraction of the WSS vector field around instantaneous WSS fixed points are observed on the aneurysmal dome and close to the neck region, reflecting the intricate hemodynamics characterizing these regions.

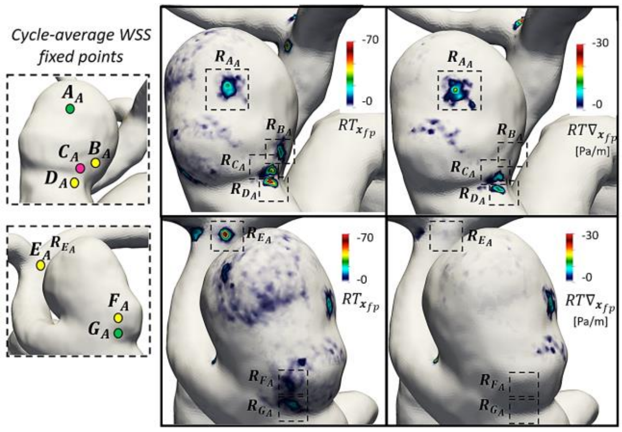

Figure 2: Maps of fixed points residence time $RT_{x_{fp}}(e)$ and of measure $RT\nabla_{x_{fp}}(e)$.

IV. Conclusion

A practical approach to analyze WSS topological skeleton was introduced and applied to computational hemodynamic models of intracranial aneurysm. The proposed approach requires WSS vector field and its divergence only and it can be easily applied to 3D vector fields defined on complex geometries. The analysis of instantaneous WSS fixed points along the cardiac cycle allows to evaluate their residence time and is the strength of local contraction/expansion using WSS divergence. Our findings on intracranial aneurysms underline the importance of focusing on instantaneous WSS fixed points analysis. The approach proposed here could contribute to facilitate and speed up studies on the significance of WSS topological skeleton in cardiovascular flows.

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